Mh4718 End-Term Study Guide

N.B.
This list also applies to the Autumn I grade exam but **does not apply** to the Autumn repeat examination.

(You will be asked to answer three out of four questions. Each question will be worth 20 marks.)

1. Write a C++ program which produces integer overflow. Explain how your program arrives at its output. (See Question 1 on Lab Sheet 2.)

2. Write a C++ program that creates two variables, x and y, both of type float and which assigns a value to x which cannot be stored exactly and assigns a machine number to y equal to the value of what is actually stored under x.

3. Determine the exact value of $\sum_{n=1}^{25} 2^{-i}$ then write a C++ program which seeks to evaluate $\sum_{n=1}^{25} 2^{-i}$ using a **float** type variable for the value of the sum. Get the program to output the value of the sum to the screen. Explain how the program arrives at this output.(See Question 1 on Worksheet 6.)

4. Create an Excel spreadsheet which demonstrates that the value which is displayed in a cell is not necessarily the value that is stored by Excel.

5. Run the following program. How can you tell that the displayed output does not accurately express the value that is stored for the variable x?

```
#include <iostream>
#include <cmath>
using namespace std;
void main()
{
    cout.precision(20);
    double x = pow(2.0,-30); \\Another number could be used here.
    cout<<x<<endl;
}
```

6. Get Excel to sketch the graph of a function and the tangent line to the graph at the point (x_0, y_0) on the same axes. The values of x_0 and y_0 respectively should be determined by values stored in two of the spreadsheet cells.
   Insert an ActiveX spinbutton control into the spreadsheet and use it to vary the value of x_0. Save as a macro enabled worksheet.(See Question 1 on Lab Sheet 5.)
7. Lab Sheet 6
8. Let R_n denote a Riemann sum which estimates a an integral $\int_a^b f(x)\mathrm{d}x$ using n equal sub-intervals of $[a, b]$.
   Write a C++ program which outputs each n and the corresponding error in R_n to a text file for $n = 2^7, 2^8, 2^9, 2^{10} \ldots, 2^{27}$.
   Use the data in the text file to plot a scattergram in MSExcel.
   Comment on what this scattergram illustrates about rounding error.
9. State and prove *Horner's method*.
10. Write a C++ program that uses Horner's method to evaluate a given polynomial at a given point. (See Question 2 Worksheet 7, Question 2 on Lab Sheet 7, Questions 1 and 2 on Lab Sheet 8.)
11. Find the Taylor polynomial of of a given degree around some point a for a given function. (See Question 1 on Worksheet 9.)
12. Write a C++ program which illustrates that Horner's method suffers from less rounding error when evaluating Taylor polynomials which estimate the function e^x. (See Question 2 Lab Sheet 10.)
13. Use a Taylor series to find the solution for a given initial value problem. (See Question 2 on Worksheet 9.)
14. Solve a given initial value problem using separation of variables. (See Question 1 on Worksheet 10 and Question 2 on Lab Sheet 11.)
15. Write a C++ program that uses Euler's method to find an approximate solution to a given initial value problem. Get the program to output the generated approximate solution values and the C++ version of the exact solution values over a given interval. (See Question 3 on Worksheet 9, Question 3 on Lab Sheet 10, Question 1 and Question 2 on Lab Sheet 11.)
16. Let F have a fixed point at p with $|F'(x)| < r < 1$ in the interval $(p - \delta, p + \delta)$ for some $\delta > 0$. Prove that the iteration with F will converge to p for any initial value $x_0 \in (p - \delta, p + \delta)$.

17. Find all fixed points for a given function $F(x)$ and determine whether or not iteration with F will converge locally to each fixed point respectively. (See Question 2 on Worksheet 10.)

18. Create a macro in MSExcel which implements a given iteration. The macro should use an ActiveX button control to move from one term of the sequence to the next. Each term of the sequence should appear, in turn, in one fixed cell and the number of iterations used to date should appear in another fixed cell with suitable labelling in each case. Save as a macro enabled worksheet.(See Question 1 and Question 2 on Lab Sheet 9.)

19. Write an Excel macro that uses Jacobi iteration in Excel to estimate a solution for a given system of linear equations. The macro should be driven by an ActiveX button control. Each click of the button should cause the next term of the iteration to be calculated. Save as a macro enabled worksheet.